Semester One Examination, 2018

Question/Answer booklet

HALE SCHOOL

Year 11
MATHEMATICS METHODS
UNIT 1

Section Two:
Calculator Allowed

Booklet 3 of 3

Student name Marking Key

Circle your teacher's Initials: **IFB** **DD** **VMU** **SWA** **MS** **AGC**

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question 16 **(4 marks)**

The set of values in the table follow the rule $y = ab^x$

x	2	4	7
y	14 400	20 736	35 831.808

(a) Determine the rule. (3 marks)

$b^2 = \frac{20736}{14400}$ $b^3 = \frac{35831.808}{20736}$ ✓ calculates b-value
$= 1.2$ $= 1.2$

Using $b = 1.2$, $14400 = a(1.2)^2$ ✓ calculates a-value
$a = 10\,000$

$\therefore$ Rule $y = 10000 \times (1.2)^x$ ✓ Gives information in a rule.

(b) Calculate the value of y when $x = 20$ (1 marks)

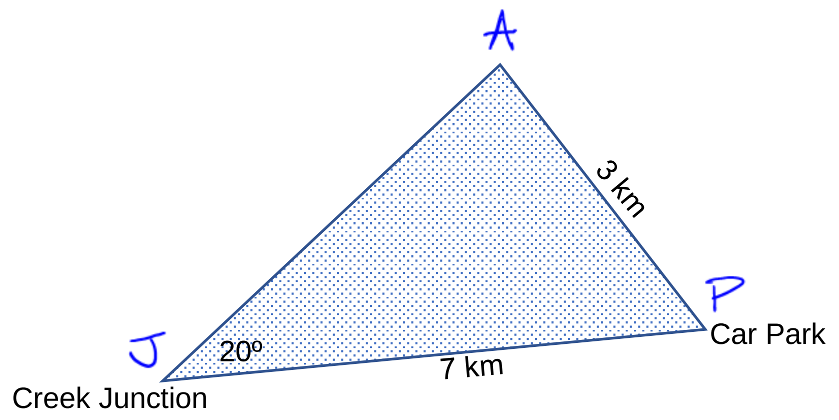
When $x = 20$

$y = 10000\,(1.2)^{20}$
$= 383\,375.9992$ (4dp) ✓ right/wrong

... according to their rule in part (a).

Question 17 **(6 marks)**

A hiker has gone missing in a National Park. The ranger provides details of the search area. The information is represented in the diagram below.



(a) Calculate the size of the search area. (5 marks)

$$\frac{3}{\sin 20^\circ} = \frac{7}{\sin A}$$ ✓ correct use of sine rule

$\therefore A = 52.94^\circ$ OR 127.06° (2dp)

When $A = 52.94^\circ$, $P = 107.06^\circ$ ✓ correct Angle P

When $A = 127.06^\circ$, $P = 32.94^\circ$ ✓ Ambiguous Case Considered.

$\therefore A(\Delta APJ) = \frac{1}{2}(7\text{km})(3\text{km})\sin(107.06^\circ)$
$= 10.04\text{ km}^2$ (2dp)
✓ correct Area

$A(\Delta APJ) = \frac{1}{2}(7\text{km})(3\text{km})\sin(32.94^\circ)$
$= 5.71\text{ km}^2$ (2dp)
✓ correct Area for second solution.

(b) Is there anything wrong with the information supplied by the park ranger? (1 mark)

Yes... Information does NOT define a unique triangle
Information is AMBIGUOUS. ✓ Valid mathematical reason ... award F/T.

Question 18 **(3 marks)**

A financial planner predicts your investment will grow at an increasing rate over time according to the rule:

$$A(t) = A_0 \times 1.047^t$$

Where $A(t)$ is the amount of your investment at the end of t years and A_0 is the amount of your initial investment.

According to this rule, what is the least number of years it will take for your investment to triple in value?

If the investment triples in value

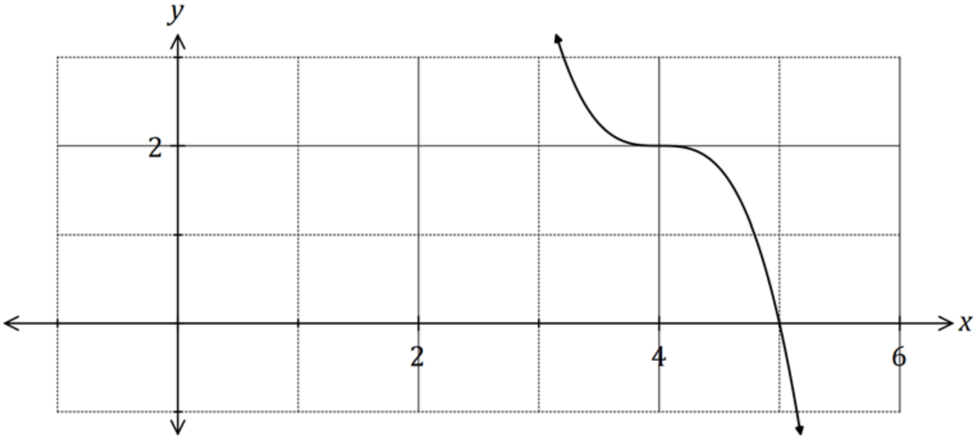
$1.047^t = 3$ ✓ Appropriate Equation to Solve.

$\therefore \quad t = 23.9$ (1dp) ✓ Value of t

$\therefore$ It will take 24 years for the investment to triple in value. ✓ Interprets answer in context

Question 19 **(6 marks)**

(a) Part of the graph of $y = f(x)$ is shown below, where $f(x) = -2(x-b)^3 + c$, and b and c are constants.



(i) State the degree of $f(x)$. (1 mark)

Degree 3. ✓ right/wrong

(ii) Determine the value of b. (1 mark)

$b = 4$ ✓ right/wrong

(iii) Determine $f(0)$. (2 marks)

$f(0) = -2(0-4)^3 + 2$ ✓ value of c

$\therefore\ f(0) = 130$ ✓ Correct value for $f(0)$.

(b) Another function is given by $g(x) = f(x+8)$.

Describe how to obtain the graph of $y = g(x)$ from the graph of $y = f(x)$. (2 marks)

Horizontal Translation ✓ correct form of transformation

8 units left ✓ How far...

Question 20 **(11 marks)**

During 2018, the altitude of the sun, A degrees, at noon in Melbourne on the n^{th} day of the year can be modelled by the equation

$$A = 23.5\,sin\left(\frac{8\pi(101+n)}{1461}\right) + 52.2$$

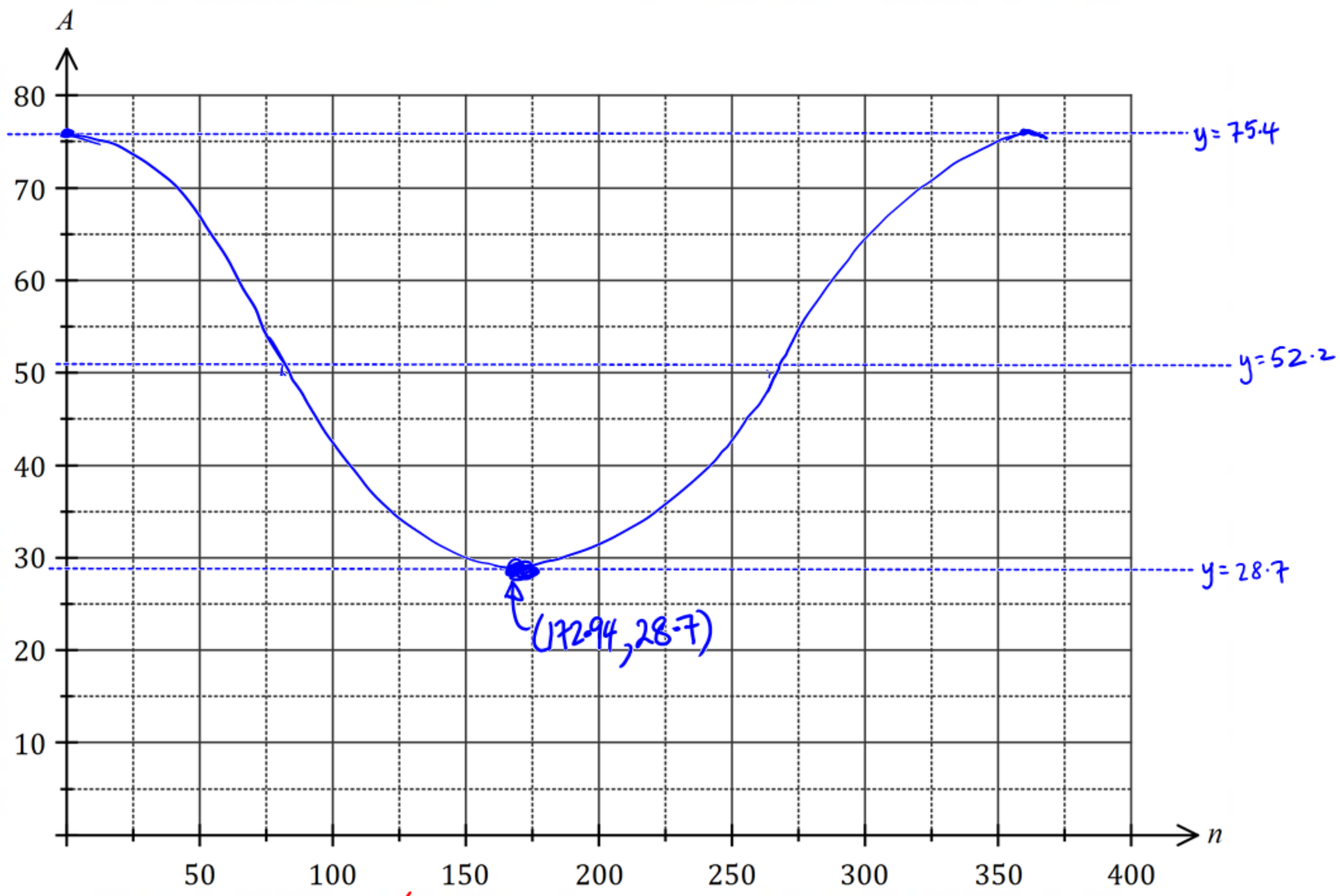
(a) On the 26th of January, the altitude of the sun was $71.4°$.
Calculate the altitude ten days earlier. (2 marks)

$n = 16$ ✓ uses $n=16$

$\therefore A = 23.5 \sin\left(\frac{8\pi(101+16)}{1461}\right) + 52.2$

$\therefore A = 73.44°$ (2dp) ✓ calculates angle correctly.

(b) Graph the altitude on the axes below for $0 \leq n \leq 365$. (4 marks)



✓ Graphs over domain $0 \leq n \leq 365$.
✓ Max Values (355.6, 75.7), $n = 365 \rightarrow A = 75.4$
✓ Min Value at $n = 172.94 \rightarrow A = 28.7$
✓ Correct Shape.

(c) State the minimum altitude of the sun at noon in Melbourne and on which day of the year this occurred. (2 marks)

✓ Altitude

Min Value 28.7 occurs when $n = 172.9375$

$\therefore$ Min Value occurs during the 173^{rd} day of the year

✓ Answer in context ... the day.

Solar panels on the roof of a Melbourne business are designed to meet its entire power needs on cloudless days when the altitude of the sun is at least $36°$ at noon.

(d) Determine the number of days the panels are expected to achieve this aim during 2018, ignoring the possibility of cloud cover. (3 marks)

$A = 36°$, When $n = 125.84 \Rightarrow n = 125$
When $n = 220.03 \Rightarrow n = 221$ } 96 days...

$\therefore$ Nº of days the panels do work $= 365 - 96$
$= 269$

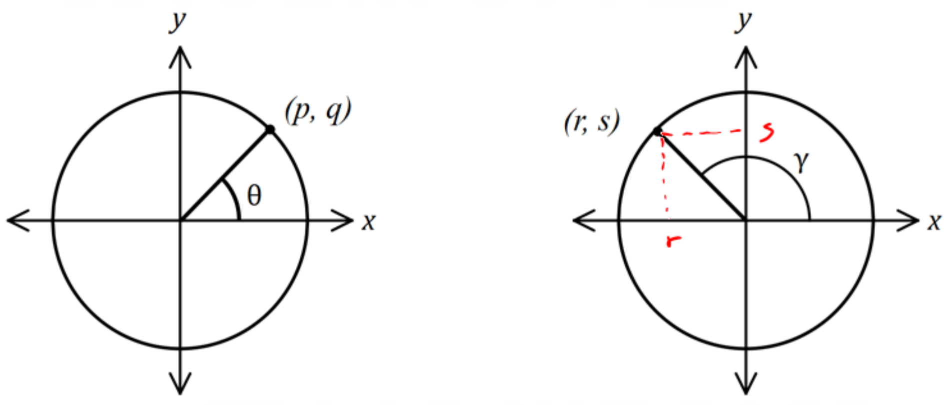
✓ Values of n when $A = 36°$

✓ Values in context of question

✓ Correct calculation of Nº of days working – For their calculations.

Question 21 **(5 marks)**

Consider the points with coordinates (p, q) and (r, s) that lie in the first and second quadrants respectively of the unit circles shown below.



Determine the following in terms of p, q, r and s, simplifying your answers where possible.

(a) $\tan\theta = \dfrac{q}{p}$ ✓ right/wrong (1 mark)

(b) $\sin(180° - \theta) = q$ ✓ right/wrong (1 mark)

(c) $\sin(\pi + \gamma) = -s$ ✓ right/wrong
. (1 mark)

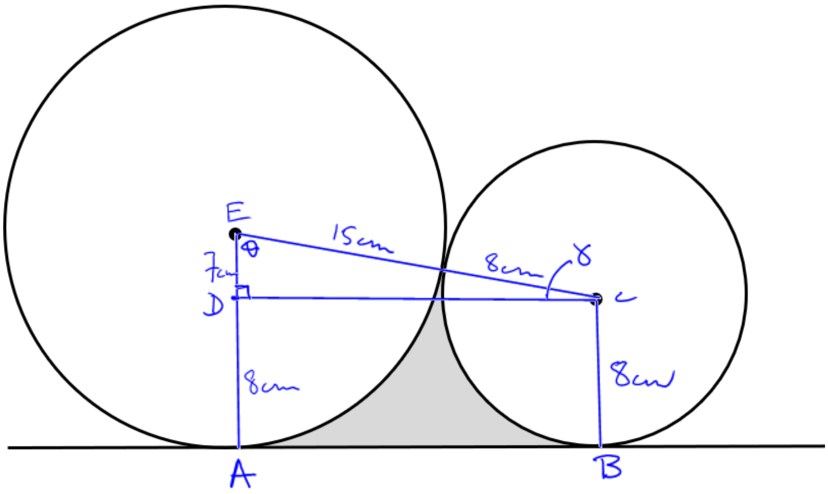
(d) $\cos(\gamma - \theta) = \cos\gamma . \cos\theta + \sin\gamma . \sin\theta$ ✓ Use of correct formula. (2 marks)

$= r . p + s . q$

$= rp + sq$ ✓ correct answer.

Question 22 **(7 marks)**

Calculate the area of the shaded region enclosed by two circles of radius 15cm and 8cm and the line, as shown in the diagram below.



In Δ CDE

$7^2 + DC^2 = 23^2$

$DC = \sqrt{480}$ ✓ calculates DC

$\cos\theta = \frac{7}{23}$ $\sin\gamma = \frac{7}{23}$

$\theta = 1.2615$ (4dp) $\gamma = 0.3093$ (4dp)

$\therefore A(\text{Large Sector}) = \frac{1}{2}(15)^2(1.2615)$ ✓ Angle for Sector

$= 141.9186\text{ cm}^2$ (4dp). ✓ Area of Sector

$A(\text{Small Sector}) = \frac{1}{2}(8)^2\left(0.3093 + \frac{\pi}{2}\right)$ ✓ Angle for Sector

$= 60.1631\text{ cm}^2$ (4dp) ✓ Area of Sector

$A(\text{Trapezium ABCE}) = \frac{1}{2}\sqrt{480} \times (15+8)$

$= 251.9524\text{ cm}^2$ (4dp) ✓ Area of Trapezium

$\therefore$ Shaded Region $= 251.9524 - 141.9186 - 60.1631$

$= 49.87\text{ cm}^2$ (2dp) ✓ Correct calculation of shaded area

End of Questions

Supplementary page

Question number: _________

Supplementary page

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